Set A

Q.12 State De'moivres theorom

→ De moivre's Theorem states that if n is any positive number/Integer then.

$\therefore r(\cos\theta + i\sin\theta)^n = r^n(\cos n\theta + i\sin n\theta)$.

→ Euler's formula representing the Complex number $\cos\theta + i\sin\theta$ is $e^{i\theta}$.

→ Solution

Let $z = -1+\sqrt{3}i \Rightarrow (x+iy)$.

Here $x = -1$ and $y = \sqrt{3}$.

To write z in polar form we note that. $\pi - \frac{\pi}{3}$.

$r = \sqrt{x^2+y^2} = \sqrt{(-1)^2+(\sqrt{3})^2} = 2$.

$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3} = \tan(180^\circ - 60^\circ) = \tan 120^\circ = \tan\frac{2\pi}{3}$.

$\therefore \theta = 2\pi/3$.

In polar form:- $z = r[\cos\theta + i\sin\theta]$

$= 2[\cos 2\pi/3 + i\sin 2\pi/3]$

To find out square root.

$z^{1/2} = \{2[\cos 2\pi/3 + i\sin 2\pi/3]\}^{1/2}$.

$= 2^{1/2}[\cos 120 \times 1/2 + i\sin 2\pi/3 \times 1/2]$ [de movire Theorem].

$= \sqrt{2}[\cos \pi/3 + i\sin \pi/3]$.

$= \sqrt{2}[\cos 60^\circ + i\sin 60^\circ]$

$= \sqrt{2}\left[\frac{1}{2} + i \times \frac{\sqrt{3}}{2}\right]$

$= \pm\left(\frac{1+i\sqrt{3}}{\sqrt{2}}\right)$

$= \frac{\pm 1}{\sqrt{2}}(1+i\sqrt{3})$.

13 (a).

Solution.

Given, α and β be the roots of $px^2+qx+q=0$.

then,

$\alpha+\beta = \frac{-b}{a} = \frac{-q}{p}$. $\alpha.\beta = \frac{c}{a} = \frac{q}{p}$.

we have.

$$\frac{\alpha+\beta}{\alpha.\beta} = \frac{-q/p}{q/p}.$$

$$\frac{\alpha+\beta}{\sqrt{\alpha.\beta}} = \frac{-q/p}{\sqrt{q/p}}.$$

$$\frac{\alpha}{\sqrt{\alpha\beta}} + \frac{\beta}{\sqrt{\alpha\beta}} = \frac{-q}{p} \times \sqrt{\frac{p}{q}}.$$

$$\frac{\sqrt{\alpha}.\sqrt{\alpha}}{\sqrt{\alpha}.\sqrt{\beta}} + \frac{\sqrt{\beta}.\sqrt{\beta}}{\sqrt{\alpha}.\sqrt{\beta}} = \frac{-\sqrt{q}.\sqrt{q}}{\sqrt{p}.\sqrt{p}} \times \frac{\sqrt{p}}{\sqrt{q}}.$$

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = -\sqrt{\frac{q}{p}}.$$

$$\therefore \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$$ proved

13 (b)

Solution.

By Inverse matrix method

Given equations, $-2x+4y=3$ and $3x-7y=1$.

Writing the system of equation in matrix form -

then $AX=C$.

$$A = \begin{pmatrix} -2 & 3 \\ 3 & -7 \end{pmatrix} X = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } C = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -2 & 3 \\ 3 & -7 \end{vmatrix} = (14-9) = 5 \neq 0.$$

A^{-1} exist.

$$A^{-1} = \frac{1}{|A|} \text{adj}. A. = \frac{1}{5}\begin{pmatrix} -7 & -3 \\ -3 & -2 \end{pmatrix}$$

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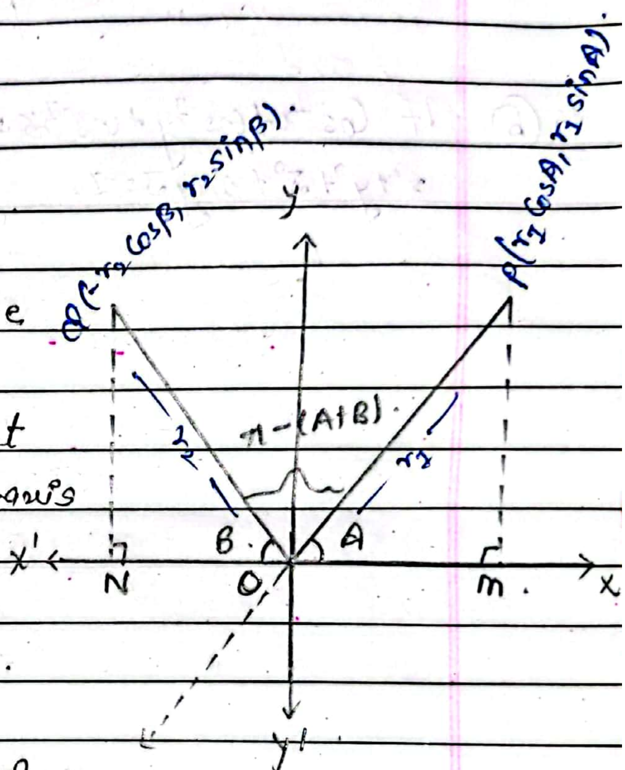
Now, $x = A^{-1}C = \frac{1}{5}\begin{pmatrix} -7 & -3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -24 \\ -11 \end{pmatrix}$

$\therefore x = \begin{pmatrix} -24/5 \\ -11/5 \end{pmatrix}$

Hen, $x = \frac{-24}{5}$ and $y = \frac{-11}{5}$.

14 (a)

Solution



proof :-
Let xox' and yoy' be the two mutually perpendicular straight line by x-axis and y-axis respectively.
Let.
$\angle xOP = A$, $\angle x'OQ = \beta$.
$\angle POQ = \pi - (A+B)$.
Also, $OP = r_1$ and $OQ = r_2$.
Also, Drawn, $MP \perp r$ on OX and $QN \perp r$ OX'.
Thus, the Coordinate of OP and OQ are $(r_1\cos A, r_1 \sin A)$ and $(-r_2\cos B, r_2\sin\beta)$.
$\therefore \vec{OP} = (r_1\cos A, r_1\sin A, 0)$
$\therefore \vec{OQ} = (-r_2\cos\beta, r_2\sin\beta, 0)$.
ie.

$$\vec{OP}\times\vec{OQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_1\cos A & r_1\sin A & 0 \\ -r_2\cos\beta & r_2\sin\beta & 0 \end{vmatrix}$$

$$= \vec{k}\,(r_1\cos A\, r_2\sin\beta + r_2\cos\beta \cdot r_1\sin A).$$

$$= \vec{k}\,(\cos A\sin\beta + \sin A\cos\beta)\, r_1 r_2.$$

$$\cancel{= \vec{k}\, r_1 r_2 \sin(A+B)}.$$

Now, if $\pi-(A+B)$ be the angle between two vector $\vec{OP}$ and $\vec{OQ}$ then,

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or, $\sin(\pi - (A+B)) = \dfrac{|\vec{OP} \times \vec{OQ}|}{|\vec{OP}| \cdot |\vec{OQ}|}$

or, $\sin(A+B) = \dfrac{r_1 r_2 (\sin A \cos B + \cos A \sin B)}{r_1 \cdot r_2}$

$\therefore \sin(A+B) = \sin A \cos B + \cos A \cdot \sin B$.

proved.

14 (b) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ then prove that $x^2+y^2+z^2+2xyz = 1$.

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Solution

Computation of Rank Correlation Coefficient.

SN.	math	Physics	Rank of math (R_1)	Rank of physics (R_2)	$d = R_1 - R_2$	d^2.
1	40	48	2.	2.	0	0
2	60	62	3	5	-2.	4
3	35	28	1	1	0	0
4	68	52	4	3.5	0.5	0.25
5	70	85	5	7	-2.	4
6	96	90	8	8	0.	0
7	70	52	6	3.5	2.5	6.25
8	84	73.	7	6	1.	1.
					$\Sigma d = 0$.	$\Sigma d^2 = 15.5$

In ascending order

for. 52

$\frac{3+4}{2} = \frac{7}{2}$

$= 3.5$

From above table

$n = 8$, $\Sigma d^2 = 15.5$, $m_1 = 2$, $R = ?$

$$R = 1 - \frac{6\left\{\Sigma d^2 + \frac{m_1(m_1^2 - 1)}{12}\right\}}{n(n^2-1)}$$

$$= 1 - \frac{6\left\{15.5 + \frac{2(2^2-1)}{12}\right\}}{8(8^2-1)}$$

$$= 1 - \frac{6\{15.5 + 0.5\}}{504}$$

$$= \frac{408}{504} = \frac{17}{21}$$

$$= 0.80.$$

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16 Using simplex method:- maximise $F = 4x - 6y$ subject to Constraints $2x - 3y \le 8$, $x + y \le 24$, $x, y \ge 0$.

Solution.

By simplex method:-

Introducing the non-negative slack variables r and s we have.

$2x - 3y + r = 8$.

$x + y + s = 24$.

Now the given lp in standard form is.

$2x - 3y + r + 0 \cdot s + 0 \cdot f = 8$.

$x + y + 0 \cdot r + s + 0 \cdot F = 24$

$-4x + 6y + 0 \cdot r + 0 \cdot s + f = 0$.

The equation in initial simplex tableau.

Basic variable	x	y	r	s	F	RHS.
r	2	-3	1	0	0	8
s.	1	1	0	1	0	24
	-4	6	0	0	1	0
	↑					

Since -4 is the most negative entry so, C_1 is pivot Column and $\frac{8}{2} < \frac{24}{1}$ so, 2 is the pivot element.

Apply $R_1 = R_1/2$.

Basic variable	x	y	r	s	F	RHS.
x	1	$-3/2$	$1/2$	0	0	4
s	1	1	0	1	0	24
	-4	6	0	0	1	0

Apply:- $R_2 = R_2 - R_1$ and $R_3 = 4R_1 + R_3$.

Basic variable	x	y	r	s	F	RHS.
x	1	$-3/2$	$1/2$	0	0	4
s.	0	$5/2$.	$-1/2$.	1	0	20.
	0	0	$1/2$	0	1	16.

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Since all the value is the last row is positive so the optimal solution is obtaine.
Hence the maximum value is 16 at $x=4$ and $y=0$

17 (a).

Solution

$$\left(\cosh\frac{x}{a}\right)^{\log x}$$

Let $y = \left(\cosh\frac{x}{a}\right)^{\log x}$

Taking log on both side,

$$\log y = \log x . \log\left(\cos{}^{hx}/_a\right).$$

$$\log y = \log\left(x + \frac{hx}{a}\right)$$

Diff. both side w.r.t. x.

or, $\frac{d(\log y)}{dy} \times \frac{dy}{dx} = \frac{d\log(x+hx/a)}{d(x+hx/a)} \times \frac{d(x+hx/a)}{dx}.$

or, $\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{x+hx/a} \times \left[1 + \frac{h}{a}\right]$

or, $\frac{dy}{dx} = y\left[\frac{a}{ax+hx} \times \frac{a+h}{a}\right]$

$\therefore \frac{dy}{dx} = \left(\cos\frac{hx}{a}\right)^{\log a} \times \frac{1}{x}.$

17 (b)

Solution

Using L-Hospital rule.

$$\lim_{x\to 0} \frac{(e^x-1)\tan x}{x^2}. \quad \left[\because \text{form } \frac{0}{0}\right]$$

18 (a) Solution

$$\int \sqrt{x^2-9}\,.dx.$$

$$= \int \sqrt{x^2-3^2}\,dx$$

$$= \frac{x\sqrt{x^2-9}}{2} - \frac{9}{2}\left(\log x + \sqrt{x^2-9}\right) + c. \quad \left[\because \int \sqrt{x^2-a^2} = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)\right]$$

$$= \frac{1}{2}\left[x\sqrt{x^2-9} - 9\log x - 9\sqrt{x^2-9}\right] + c.$$

(b) $$\int \frac{1}{(a^2+x^2)(b^2+x^2)}\,dx$$

Solution

$$\int \frac{1}{(a^2+x^2)(b^2+x^2)}\,dx$$

put $y = x^2$.

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Solution

↳. Consumer Surplus is defined as the difference between the expenditure which a Consumer is ready to pay for the use of goods ~~from~~ and the actual amount paid for Q_0 units of goods at present market price P_0 per unit.

Symbolically:- $C.S. = \int_0^{Q_0} (\text{demand function})\, dQ - P_0 Q_0.$

↳. The difference between the total revenue actually received and the total revenue that would have been willing to receive is known as producer surplus.

$P.S. = P_0 Q_0 - \int_0^{Q_0} P . dQ.$

Solution

Demand function $(P_d) = 16 - x^2$

Suppy function $(P_s) = 2x^2 + 4.$

For pure Compedition market.

$P_d = P_s.$

or, $16 - x^2 = 2x^2 + 4.$

or, $12 = 3x^2$

or, $x^2 = 4$

or, $x = \pm 2$

$\therefore x = 2$ (-2 is invalid becaus quantity is positive)

Thus, the market price is $P = 16 - x^2$

$= 16 - 2^2 = \text{Rs}. 12.$

i.e. $(P, x) = (12, 2).$

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(i) Consumer Surplus (CS) $= \int_0^2 (16-x^2)\,dx - 2\times 12.$

$$= \left[16x - \frac{x^3}{3}\right]_0^2 - 24$$

$$= \left[16\times 2 - \frac{8}{3}\right] - 24$$

$$= \frac{16}{3}.$$

(ii) Producer Surplus (PS) $= 2\times 12 - \int_0^2 (2x^2+4)\,dx.$

$$= 24 - \left[\frac{2x^3}{3} + 4x\right]_0^2.$$

$$= 24 - \left[\frac{2(2)^3}{3} + 4\times 2 - 0\right]$$

$$= 24 - \left[\frac{16}{3} + 8\right]$$

$$= \frac{32}{3}.$$

Group C.

2. (a) Find the general term and sum upto n terms of the series. $1\cdot n + 2\cdot(n-1) + 3(n-2) + \ldots\ldots$

Solution

$\Rightarrow n-1-n.$
$\Rightarrow -1r.$
$\Rightarrow -r+n+1$

Let r^{th} term of $1, 2, 3, \ldots\ldots = r.$

$t_n = n-r+1$

The r^{th} term of $n, (n-1), (n-2), \ldots\ldots = n-r+1.$

$t_1 = n-1+1 = n$
$t_2 = n-2+1 = n-1$

Now, rth term of given series be,

$\therefore t_r = r(n-r+1). = nr - r^2 + r.$ [general term].

Now,

$S_n = \sum_1^n t_r = \sum nr - \sum r^2 + \sum r.$ Now express r in term of n.

$$= n\cdot\frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}.$$

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$$= \frac{n(n+1)[3n-2n-1+3]}{6}$$

$$\therefore S_n = \frac{n(n+1)[n+2]}{6}.$$

(b) State binomial theoram. If

Solution

→ Binomial theoram States that any expression of the form $x+a$ or $ax+b$ etc. i.e. an expression Containing two terms Called binomial which Can express any power of binomial expression as a series.

Symboically:-

$$(a+x)^n = c(n,0)a^n + c(n,1)a^{n-1}x + c(n,2)a^{n-2}x^2 + \cdots + c(n,n)x^n$$

$$(a-x)^n = c(n,0)a^n - c(n,1)a^{n-1}x + c(n,2)a^{n-2}x^2 - \cdots + (-1)^n c(n,n)x^n$$

Solution

Given,

$$(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n \text{ ------ (i)}$$

$$(1+x)^n = C_nx^n + C_{n-1}x^{n-1} + C_2x^{n-2} + \cdots + C_1x + C_0 \text{ ---- (ii)}$$

multiplying eq^n (i) and (ii) we get.

$$(1+x)^{2n} = C_0C_nx^n + C_1C_{n-1}x^n + C_2C_{n-2}x^n + \cdots + C_nC_0x^n.$$

$$(1+x)^{2n} = x^n[C_0C_n + C_1C_{n-1} + C_2C_{n-2} + \cdots + C_nC_0] \text{ --- (iii)}$$

Since equation (iii) is an identity If the Coefficient of x^n in LHS is equal to the Coefficient of x^n in RHS.

The Coefficient of x^n in LHS of (iii).

$$(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1x + \cdots + {}^{2n}C_nx^n + \cdots.$$

i.e. ${}^{2n}C_n$ ------ (iv).

Equating the Coefficient of x^n in eq^n (iii) and (iv).

$$C_0C_n + C_1C_{n-1} + \cdots + C_nC_0 = {}^{2n}C_n = \frac{2n!}{(2n-n)!\,n!}$$

$$\therefore C_0C_n + C_1C_{n-1} + C_2C_{n-2} + \cdots + C_nC_0 = \frac{2n!}{n!\,n!}. \text{ Proved}$$

Q.1 (a) Define

Ans: → If α, β, γ be the angle made by a line with positive x-axis, y-axis and z-axis respectively. Then the Cosines of these angles are known as direction Cosines of a line. ie. $\cos\alpha, \cos\beta, \cos\gamma$ are the dcs. which are usually dnoted by l, m, n.

(i) → The direction of of Coordinate axes as:-

axis	dcs.
$Ox \to$ x-axis	$(1,0,0)$
$Oy \to$ y-axis	$(0,1,0)$
$Oz \to$ z-axis	$(0,0,1)$.

(ii)

axes.	Projections of AB on axes.
Ox	$1(x_1-x_2)+0(y_1-y_2)+0(z_1-z_2) \Rightarrow x_1-x_2$
Oy	$0+1(y_1-y_2)+0 \Rightarrow y_1-y_2$.
Oz	$0+0+1(z_1-z_2) \Rightarrow z_1-z_2$.

(iii) Square of length of $AB = \left[\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}\right]^2$

$$= (x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2.$$

Now, sum of square of projection are:-

$$(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2$$

proved.

(b) (d)

Solution

Given, Ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a>b>0$.

(i) Coordinate of vertices is $(\_a,0)$.

(ii) " " foci is $(\pm ae,0)$.

(iii) Equation of diretix is $x=\pm$ _.

22 (a)

Solution

Given,

$$f(x)=x(x-2)$$
$$=x^2-2x$$

$\therefore$ $f(x)$ is a polynomial function, so it is Continuous in $[1,4]$.

Again $f'(x)=2x-2$ which exists for all $x\in(1,4)$ so, it is differential in $(1,4)$.

Hence the Condition of mean value theorom are satisfied so there exist at least one ele value c in $(1,4)$ such that $f'(c)=\frac{f(b)-f(a)}{b-a}$.

But $f(b)=f(4)=4^2-2\times4=16-8=8$.

$f(a)=f(1)=1^2-2\times1=-1$.

$$f'(c)=2c-2=\frac{8-(-1)}{4-1}.$$

$$\text{or, } 2c-2=\frac{9}{3}$$

$$\therefore c=5/2\in(1,4)$$

Hence, mean value theorom is satisfied.

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(b)

Solution

Given,

$$\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}.$$

(i) It is homogeneous differential equation and it's order is 1.

(ii) Solution

Put $y = vx$ i.e. $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Now, $v + x\frac{dv}{dx} = \frac{vx}{x} - \sin^2\frac{vx}{x}$.

$$\cancel{v} + x\frac{dv}{dx} = \cancel{v} - \sin^2 v.$$

$$\frac{dv}{\sin^2 v} = \frac{-dx}{x}.$$

On Integration.

or, $\int \frac{dv}{\sin^2 v} = -\int \frac{dx}{x}$.

or, $\int \operatorname{Cosec}^2 v \cdot dv = -\int \frac{dx}{x}$

or, $\operatorname{Cot} v = -\ln x + \ln c$.

or, $\operatorname{Cot} v = \ln\left(\frac{c}{x}\right)$.

or, $\frac{c}{x} = e^{\operatorname{Cot} v}$.

$\therefore C = x e^{\operatorname{Cot}(y/x)}$. #